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## Research and Development Technical Report

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### ANALYSIS OF TARGET POSITION ERROR ASSOCIATED WITH AN AIRBORNE SCOUT HELICOPTER

H. Gorman

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Avionics Laboratory

June 1976

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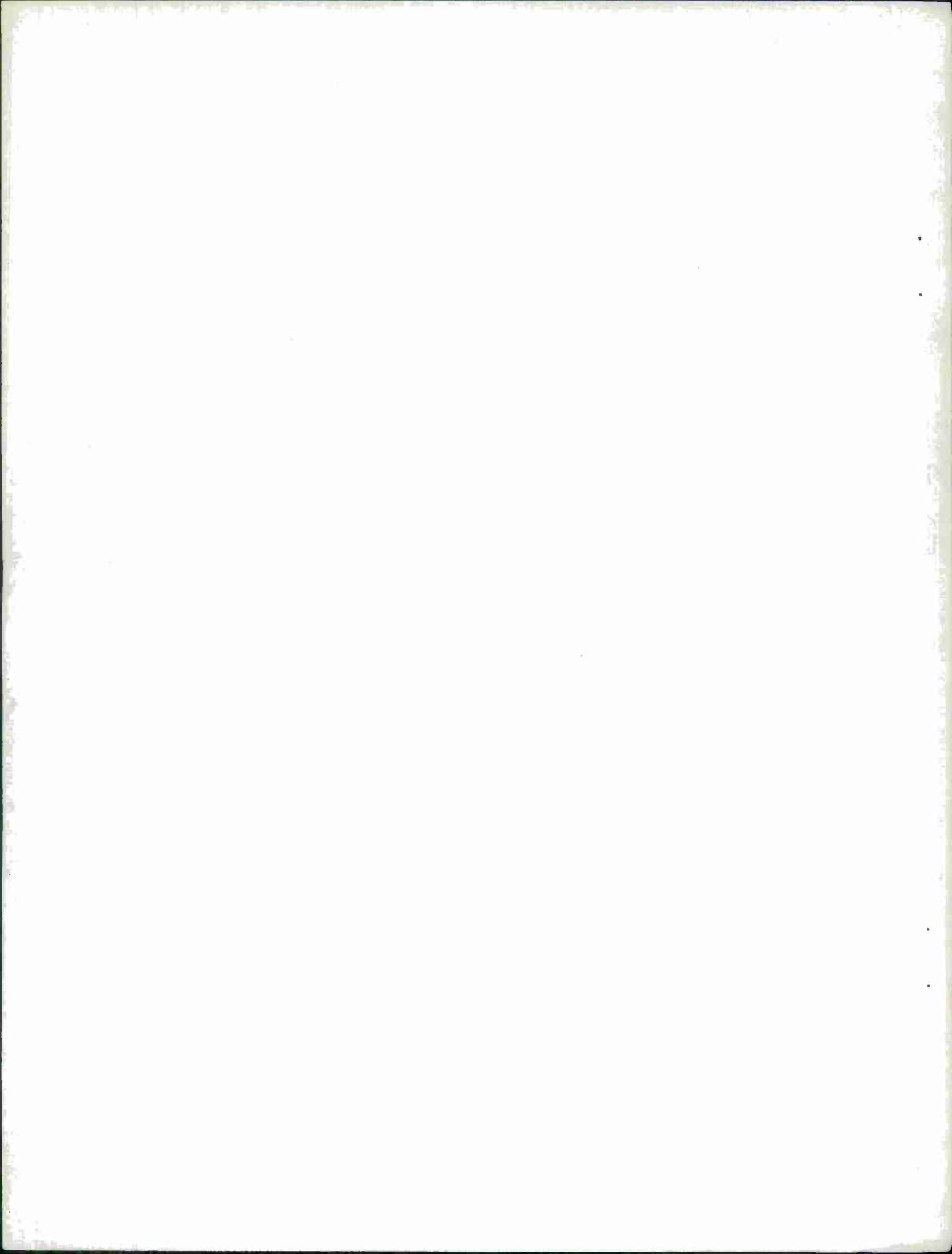
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## SUMMARY

When configuring an avionics system for a Scout helicopter, one must determine the extent to which the on-board avionics equipment impacts the target location accuracy.

In this report, a mathematical error model defining a typical geometry and relevant parameters associated with locating a target on the ground from an airborne Scout helicopter has been developed. Analysis of the model has demonstrated that the major contributions to target location error are navigation error and heading error, and that all other errors considered are relatively insignificant.



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## 1. DERIVATION OF TARGET POSITION EQUATIONS

Before an analysis of target position error can be performed, the equations defining the problem geometry must be derived. In the following derivation, the target position is described as a function of those parameters which are both pertinent to the problem geometry and capable of being measured by on-board equipment in the Scout helicopter. The following parameters are considered: Scout UTM coordinates, Scout altitude, Scout heading, Scout attitude with respect to local vertical, azimuth and elevation angles of the target relative to the Scout body axis, and slant range from the Scout to the target.

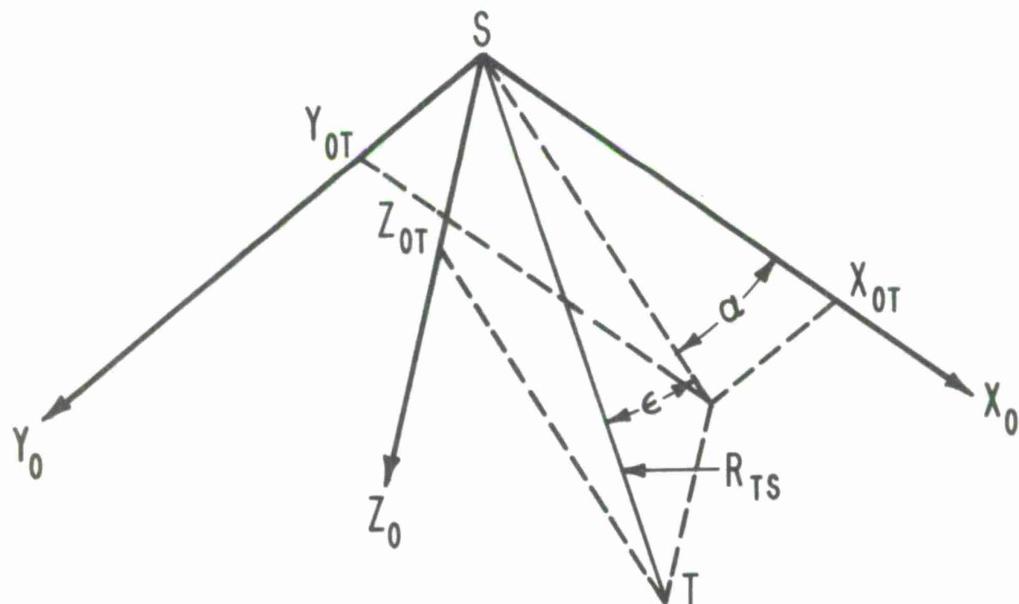


Figure 1. Definition of relative Scout-target parameters

Let the axes  $X_0$ ,  $Y_0$ , and  $Z_0$  represent the body axes of the Scout aircraft (S). From Figure 1, we have

$$X_{OT} = R_{TS} \cdot \cos \epsilon \cdot \cos \alpha \quad 1a$$

$$Y_{OT} = R_{TS} \cdot \cos \epsilon \cdot \sin \alpha \quad 1b \quad (1)$$

$$Z_{OT} = R_{TS} \cdot \sin \epsilon \quad 1c$$

where  $X_{OT}$ ,  $Y_{OT}$ , and  $Z_{OT}$  are the target (T) coordinates in the XYZ system,  $\alpha$  and  $\epsilon$  are the relative azimuth and elevation, respectively, of the target with respect to the Scout's body axis and  $R_{TS}$  is the slant range between the Scout and target. Consider now the Scout's attitude with respect to a horizontal reference system  $X_3$ ,  $Y_3$ ,  $Z_3$  with  $X_3$  pointing North,  $Y_3$  pointing East, and  $Z_3$  pointing vertically down. Take first, a rotation through an angle  $H$  (heading) about the  $Z_3$  axis as illustrated in Figure 2.

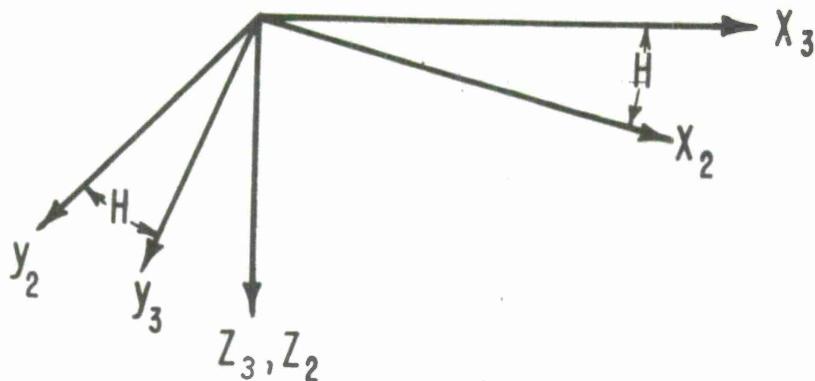


Figure 2. Heading angle rotation

Then

$$\begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix} = \begin{bmatrix} \cos H & -\sin H & 0 \\ \sin H & \cos H & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} \quad (2)$$

Second, perform a rotation through an angle E (elevation) about the Y2 axis, as illustrated in Figure 3.

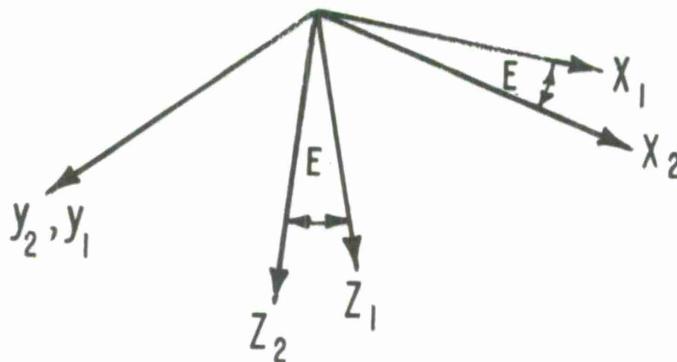


Figure 3. Elevation angle rotation

then

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \cos E & 0 & \sin E \\ 0 & 1 & 0 \\ -\sin E & 0 & \cos E \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} \quad (3)$$

Finally, a rotation about an angle  $\phi$  (roll) is performed about the  $X_1$  axis as illustrated in Figure 4.

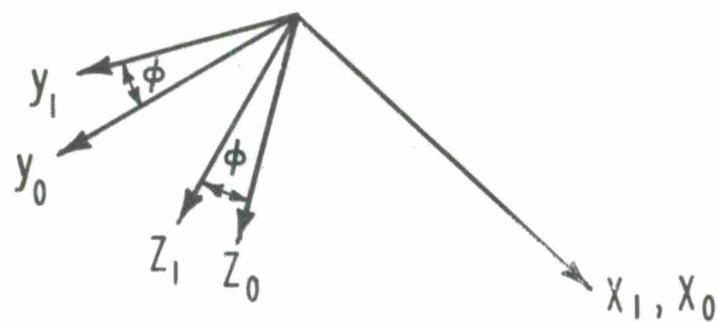


Figure 4. Roll angle rotation

Then

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} \quad (4)$$

Combining equations 2, 3, and 4:

$$\begin{bmatrix} X_{3T} \\ Y_{3T} \\ Z_{3T} \end{bmatrix} = \begin{bmatrix} CH & -SH & 0 \\ SH & CH & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} CE & 0 & SE \\ 0 & 1 & 0 \\ -SE & 0 & CE \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\phi & -S\phi \\ 0 & S\phi & C\phi \end{bmatrix} \begin{bmatrix} X_{OT} \\ Y_{OT} \\ Z_{OT} \end{bmatrix} \quad (5)$$

where CH denotes  $\cos(H)$ , etc., for brevity. Multiplying the matrices and substituting equations 1a, 1b, and 1c into (5) yields

$$\begin{bmatrix} X_{3T} \\ Y_{3T} \\ Z_{3T} \end{bmatrix} = \begin{bmatrix} CHCE & CHSES\phi & CHSEC\phi \\ SHCE & SHSES\phi & SHSEC\phi \\ -SE & CES\phi & CEC\phi \end{bmatrix} \begin{bmatrix} R_{TS} & C\varepsilon Ca \\ R_{TS} & C\varepsilon Sa \\ R_{TS} & S\varepsilon \end{bmatrix} \quad (6)$$

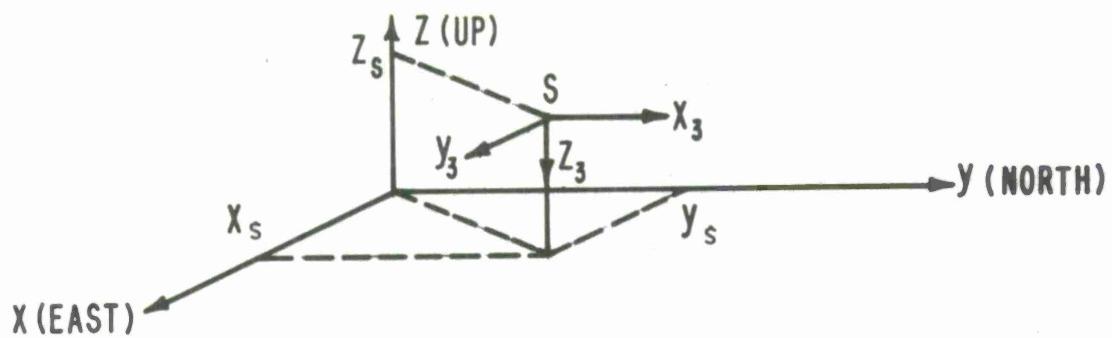


Figure 5. Definition of ground coordinate system

If the  $X_3, Y_3, Z_3$  coordinate system is now referenced to a horizontal ground coordinate system,  $X, Y, Z$  with  $X$  East,  $Y$  North, and  $Z$  up, as shown in Figure 5 there results

$$X_T = X_S + Y_{3T} \quad 7a$$

$$Y_T = Y_S + X_{3T} \quad 7b \quad (7)$$

$$Z_R = Z_S - Z_{3T} \quad 7c$$

where  $X_T, Y_T, Z_T$  are the target coordinates and  $X_S, Y_S, Z_S$  are the Scout coordinates in the  $X, Y, Z$  system. Combining equations (7a), (7b), and (7c) with equation (6) yields

$$\begin{bmatrix} Y_T \\ X_T \\ -Z_T \end{bmatrix} = \begin{bmatrix} Y_S \\ X_S \\ -Z_S \end{bmatrix} + R_{TS} \begin{bmatrix} CHCE & +CHSES\phi & +CHSEC\phi \\ SHCE & +SHSES\phi & +SHSEC\phi \\ -SE & CES\phi & CEC\phi \end{bmatrix} \begin{bmatrix} C\epsilon C\alpha \\ C\epsilon S\alpha \\ S\epsilon \end{bmatrix} \quad (8)$$

Assuming that  $X$  and  $Y$  represent UTM coordinates and  $Z$  represents altitude, we now have an equation defining the target location as a function of

- Scout UTM coordinates  $X_S$  and  $Y_S$
- Scout altitude  $Z_S$

- Scout heading, H
- Scout attitude with respect to local vertical (elevation, E and roll,  $\phi$ )
- Azimuth angle,  $\alpha$ , of the target relative to the Scout
- Elevation angle,  $\varepsilon$ , of the target relative to the Scout
- Slant range,  $R_{TS}$  from the Scout to the target

i.e., Target Location =  $f(X_s, Y_s, Z_s, H, E, \phi, \alpha, \varepsilon, R_{TS})$

In component form, equation (8) becomes

$$X_T = X_s + R_{TS} \{ SH [C\varepsilon(CEC\alpha + SES\phi S\alpha) + SEC\phi S\varepsilon] \\ + CH(C\phi C\varepsilon S\alpha - S\phi S\varepsilon) \} \quad 9a$$

$$Y_T = Y_s + R_{TS} \{ CH [C\varepsilon(CEC\alpha + SES\phi S\alpha) + SEC\phi S\varepsilon] \\ - SH(C\phi C\varepsilon S\alpha - S\phi S\varepsilon) \} \quad 9b \quad (9)$$

$$Z_T = Z_s + R_{TS} \{ C\varepsilon(SEC\alpha - CES\phi S\alpha) - CEC\phi S\varepsilon \} \quad 9c$$

It is worth noting here that equation (8) may be rewritten as follows:

$$\frac{1}{R_{TS}} \begin{bmatrix} Y_T - Y_s \\ X_T - X_s \\ -(Z_T - Z_s) \end{bmatrix} = M \begin{bmatrix} C\varepsilon C\alpha \\ C\varepsilon S\alpha \\ S\varepsilon \end{bmatrix}, \text{ M = matrix in equation (8)}$$

or

$$\frac{1}{R_{TS}} M^T \begin{bmatrix} Y_T - Y_s \\ X_T - X_s \\ (Z_T - Z_s) \end{bmatrix} = \begin{bmatrix} C\varepsilon C\alpha \\ C\varepsilon S\alpha \\ S\varepsilon \end{bmatrix} \quad (8a)$$

Then, if we pick  $X_s, Y_s, Z_s, X_T, Y_T, Z_T, H, E$ , and  $\phi$ , the variable  $R_{TS}$  can be calculated as  $R_{TS} = [(X_T - X_s)^2 + (Y_T - Y_s)^2 + (Z_T - Z_s)^2]^{1/2}$  and  $\alpha$  and  $\varepsilon$  can be determined by equation (8a) above.

## 2. DERIVATION OF ERROR EQUATIONS

A linear error analysis was performed on the target position variables  $X_T$ ,  $Y_T$ , and  $Z_T$ . Considering, first,  $X_T$ , we write

$$X_T = X_T (X_S, Y_S, Z_S, H, E, \phi, \alpha, \epsilon, R_{TS})$$

letting

$$\xi_1 = X_S$$

$$\xi_2 = Y_S$$

$$\xi_3 = Z_S$$

$$\xi_4 = H$$

$$\xi_5 = E$$

$$\xi_6 = \phi$$

$$\xi_7 = \alpha$$

$$\xi_8 = \epsilon$$

$$\xi_9 = R_{TS}$$

and taking the total derivative of  $X_T$  yields

$$dx_T = \sum_{i=1}^9 \frac{\partial X_T}{\partial \xi_i} d\xi_i$$

we now define  $d_x_T = E_x$ , the error in the computed value of the target coordinate and  $d\xi_i = e_i$ , the error in the measurement of the  $\xi_i$ th variable.

Then

$$E_x = \sum_{i=1}^9 \frac{\partial X_T}{\partial \xi_i} e_i \quad 10a$$

Similarly

$$E_y = \sum_{i=1}^9 \frac{\partial y_T}{\partial \xi_i} e_i \quad 10b \quad (10)$$

$$E_z = \sum_{i=1}^9 \frac{\partial z_T}{\partial \xi_i} e_i \quad 10c$$

Calculate first the partial derivatives of  $X_T$  with respect to the  $\xi_i$ :

1.  $\partial X_T / \partial X_S = 1$
2.  $\partial X_T / \partial Y_S = 0$
3.  $\partial X_T / \partial Z_S = 0$
4.  $\partial X_T / \partial H = R_{TS} \{ CH[C\epsilon(CEC\alpha + SES\phi S\alpha) + SEC\phi S\epsilon] - SH(C\phi C\epsilon S\alpha - S\phi S\epsilon) \}$

but, inspection of (9b) shows this is just  $Y_T - Y_S$

$$\frac{\partial X_T}{\partial H} = Y_T - Y_S$$

5.  $\partial X_T / \partial E = R_{TS} \{ SH[C\epsilon (SEC\alpha - CES\phi S\alpha) - CEC\phi S\epsilon] \}$

but, inspection of (9c) shows this is just  $SH(Z_T - Z_S)$

$$\partial X_T / \partial E = SH(Z_T - Z_S)$$

6.  $\partial X_T / \partial \phi = R_{TS} \{ SH [C\epsilon SEC\phi S\alpha - SES\phi S\epsilon] + CH [-S\phi C\epsilon S\alpha - C\phi S\epsilon] \}$
7.  $\partial X_T / \partial \alpha = R_{TS} \{ SH[CE(-CES\alpha + SES\phi C\alpha)] + CH C\phi C\epsilon C\alpha \}$
8.  $\partial X_T / \partial \epsilon = R_{TS} \{ SH[-S\epsilon(CEC\alpha + SES\phi S\alpha) + SEC\phi C\epsilon] + CH (-C\phi S\epsilon S\alpha - S\phi C\epsilon) \}$
9.  $\partial X_T / \partial R_{TS} = \{ SH[(CEC\alpha + SES\phi S\alpha)C\epsilon + SEC\phi S\epsilon] + CH[C\phi C\epsilon S\alpha - S\phi S\epsilon] \}$

but, inspection of (9a) shows this is just  $(X_T - X_S) / R_{TS}$

$$\frac{\partial X_T}{\partial R_{TS}} = (X_T - X_S) / R_{TS}$$

Next, calculate the partial derivatives of  $Y_T$  with respect to the  $\xi_i$ :

1.  $\partial Y_T / \partial X_S = 0$
2.  $\partial Y_T / \partial Y_S = 1$
3.  $\partial Y_T / \partial Z_S = 0$
4.  $\partial Y_T / \partial E = -(X_T - X_S)$
5.  $\partial Y_T / \partial E = CH(Z_T - Z_S)$
6.  $\partial Y_T / \partial \phi = R_{TS} \{ CH [C\epsilon SEC\phi S\alpha - SES\phi S\epsilon] - SH [-S\phi C\epsilon S\alpha - C\phi S\epsilon] \}$
7.  $\partial Y_T / \partial \alpha = R_{TS} \{ CH[C\epsilon (-CES\alpha + SES\phi C\alpha)] - SH(C\phi C\epsilon C\alpha) \}$
8.  $\partial Y_T / \partial \epsilon = R_{TS} \{ CH[-S\epsilon(CEC\alpha + SES\phi S\alpha) + SEC\phi C\epsilon] - SH[-C\phi S\epsilon S\alpha - S\phi C\epsilon] \}$
9.  $\partial Y_T / \partial R_{TS} = (Y_T - Y_S) / R_{TS}$

Finally, calculate the partial derivatives of  $Z_T$  with respect to the  $\xi_i$ :

1.  $\partial Z_T / \partial X_S = 0$
2.  $\partial Z_T / \partial Y_S = 0$
3.  $\partial Z_T / \partial Z_S = 1$
4.  $\partial Z_T / \partial H = 0$
5.  $\partial Z_T / \partial E = -SH(X_T - X_S) + CH(Y_T - Y_S)$
6.  $\partial Z_T / \partial \phi = R_{TS} \{ C\varepsilon (CEC\phi S\alpha) - CES\phi S\varepsilon \}$
7.  $\partial Z_T / \partial \alpha = -R_{TS} C\varepsilon (CES\phi C\alpha + SES\alpha)$
8.  $\partial Z_T / \partial \varepsilon = -R_{TS} \{-S\varepsilon (CES\phi S\alpha - SEC\alpha) + CEC\phi C\varepsilon \}$
9.  $\partial Z_T / \partial R_{TS} = (Z_T - Z_S) / R_{TS}$

Letting

$$X_{TS} = Y_T - X_S$$

$$Y_{TS} = Y_T - Y_S$$

$$Z_{TS} = Z_T - Z_S$$

There results

$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & Y_{TS} & (CH)Z_{TS} & \frac{\partial X_T}{\partial \phi} & \frac{\partial X_T}{\partial \alpha} & \frac{\partial X_T}{\partial \varepsilon} & \frac{X_{TS}}{R_{TS}} \\ 0 & 1 & 0 & -X_{TS} & (SH)Z_{TS} & \frac{\partial Y_T}{\partial \phi} & \frac{\partial Y_T}{\partial \alpha} & \frac{\partial Y_T}{\partial \varepsilon} & \frac{Y_{TS}}{R_{TS}} \\ 0 & 0 & 1 & 0 & -(SH)X_{TS} & \frac{\partial Z_T}{\partial \phi} & \frac{\partial Z_T}{\partial \alpha} & \frac{\partial Z_T}{\partial \varepsilon} & \frac{Z_{TS}}{R_{TS}} \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_z \\ e_H \\ e_E \\ e_\phi \\ e_\alpha \\ e_\varepsilon \\ e_R \end{bmatrix}$$

(11)

The matrix equation 11 defines the total error in target position as a function of the nine Scout-target variables  $X_s$ ,  $Y_s$ ,  $Z_s$ ,  $H$ ,  $E$ ,  $\phi$ ,  $\alpha$ ,  $\epsilon$ , and  $R_{TS}$  and the nine associated errors,  $e_i$ .

### 3. ANALYSIS OF THE ERROR EQUATIONS

Assume that the nine Scout-target variables have been assigned specific values and, therefore, the matrix elements in equation (11) have been fixed. Let these elements be denoted  $C_{ij}$  ( $i$  = row;  $j$  = column). Next, assume that the  $e_i$  are independent, normally distributed random variables with density functions.

$$f_i(\eta) = N[\mu_i, \sigma_i] = \frac{1}{\sigma_i \sqrt{2\pi}} e^{-(\eta - \mu_i)^2 / 2\sigma_i^2}$$

It can then be shown that the density functions corresponding to the random variables

$$E_x = \sum_{j=1}^9 C_{1j} e_j$$

$$E_y = \sum_{j=1}^9 C_{2j} e_j$$

$$E_z = \sum_{j=1}^9 C_{3j} e_j$$

are normal with means

$$\bar{\mu}_k = \sum_{j=1}^9 C_{kj} \mu_j \quad k = 1, 2, 3$$

and standard deviations

$$\bar{\sigma}_k = \left( \sum_{j=1}^9 (C_{kj} \sigma_j)^2 \right)^{1/2} \quad k = 1, 2, 3$$

That is, the normal density functions corresponding to  $E_x$ ,  $E_y$ , and  $E_z$  are

$$f_{E_x} = N \left\{ \sum_{j=1}^9 C_{ij} \mu_j, \left[ \sum_{j=1}^9 (C_{ij} \sigma_j)^2 \right]^{1/2} \right\}$$

$$f_{E_y} = N \left\{ \sum_{j=1}^9 C_{2j} \mu_j, \left[ \sum_{j=1}^9 (C_{2j} \sigma_j)^2 \right]^{1/2} \right\}$$

$$f_{E_z} = N \left\{ \sum_{j=1}^9 C_{3j} \mu_j, \left[ \sum_{j=1}^9 (C_{3j} \sigma_j)^2 \right]^{1/2} \right\}$$

Suppose, now, that it is desired to perform the above analysis for many Scout-target states. For simplicity, only the distributions in  $E_x$  will be considered. For  $N$  Scout-target states, there results  $N$  normally independent probability distribution functions (pdf)  $f_i$ , with corresponding means  $\mu_i$ ; and standard deviations  $\sigma_i$ ,  $i = 1, 2, \dots, N$ . We now pose the following question:

If a sample is taken at random (with equal likelihood) from one of the distributions, what is the distribution of the sample?

The sample space is shown in Figure 6.

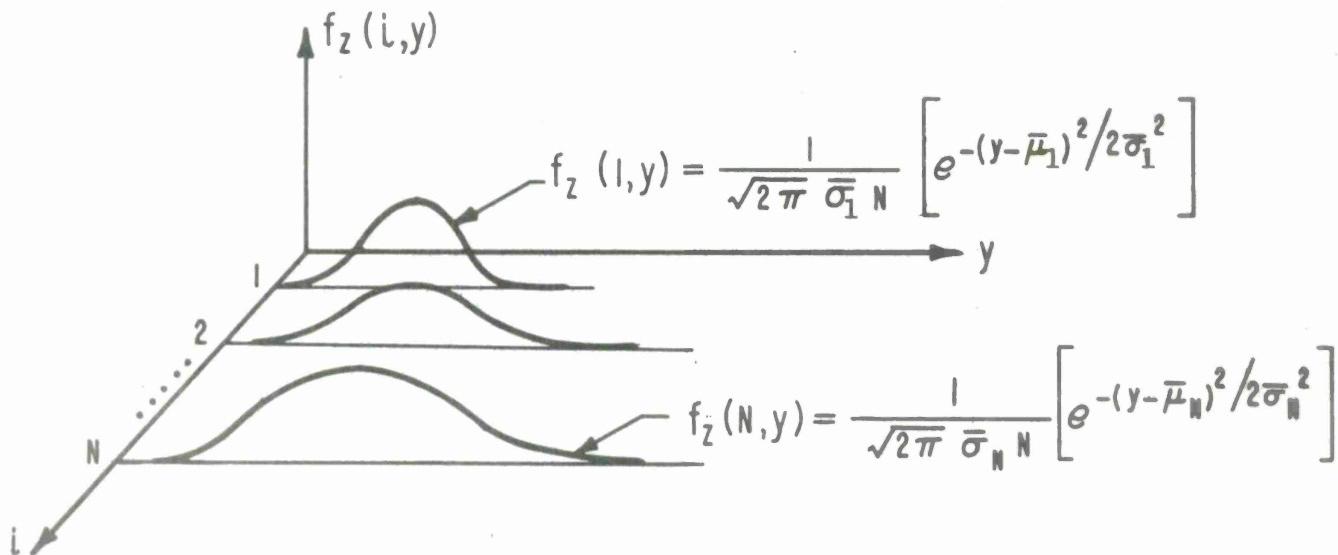


Figure 6.  $N$  normally independent probability distribution functions comprising sample space

Since the sample is taken with equal likelihood, the sample space is composed of N functions of the form:

$$f_z(i, y) = \frac{1}{\sqrt{2\pi} \sigma_i N} \frac{-(y - \bar{\mu}_i)^2}{2\sigma_i^2}$$

$$i = 1, 2, \dots, N$$

Summing over i yields the pdf of the sample:

$$f_z(y) = \frac{1}{\sqrt{2\pi} N} \sum_{i=1}^N \frac{e^{-(y - \bar{\mu}_i)^2 / 2\sigma_i^2}}{\sigma_i}$$

Then the mean of the sample may be calculated as

$$\mu_z = \int_{-\infty}^{\infty} z f_z(z) dz$$

$$= \frac{1}{N} \sum_{i=1}^N \frac{1}{\sqrt{2\pi} \sigma_i} \int_{-\infty}^{\infty} z e^{-(z - \bar{\mu}_i)^2 / 2\sigma_i^2} dz$$

$$\mu_z = \frac{1}{N} \sum_{i=1}^N \bar{\mu}_i$$

and the variance as

$$\sigma_z^2 = \int_{-\infty}^{\infty} z^2 f_z(z) dz - \mu_z^2$$

$$\sigma_z^2 = \frac{1}{N} \sum_{i=1}^N \frac{1}{\sqrt{2\pi} \sigma_i} \int_{-\infty}^{\infty} z^2 e^{-(z - \bar{\mu}_i)^2 / 2\sigma_i^2} dz - \mu_z^2$$

$$\sigma_z^2 = \frac{1}{N} \sum_{i=1}^N (\bar{\sigma}_i^2 + \bar{\mu}_i^2) - \mu_z^2$$

#### 4. RESULTS

Several techniques were used in performing the analysis. The first consisted of choosing nominal values for  $X_s$ ,  $Y_s$ ,  $Z_T$ , and  $H$  and allowing  $X_T$ ,  $Y_T$ ,  $Z_S$ ,  $E$ , and  $\phi$  to vary in increments over predetermined ranges.  $R_{TS}$ ,  $\alpha$  and  $\epsilon$  were calculated according to equation (8a). For convenience,  $X_T$  and  $Y_T$  were transformed into polar coordinates by the equations:

$$X_T = r \sin(\theta)$$

$$Y_T = r \cos(\theta)$$

and then the parameters  $r$  and  $\theta$  were varied. The following tables list the standard deviations corresponding to each  $e_i$  and the maximum, minimum, and incremental values of the Scout-target parameters.

TABLE 1. STANDARD DEVIATIONS AND MEANS CORRESPONDING TO EACH VARIABLE,  $e_i$

error	$\sigma$	$\mu$
$e_x$	55.0 m	0
$e_y$	55.0 m	0
$e_z$	2.0°	0
$e_H$	1.0°	0
$e_E$	0.5°	0
$e_\phi$	0.5°	0
$e_\alpha$	0.2 mrad	0
$e_\varepsilon$	0.2 mrad	0
$e_R$	10.0 m	0

TABLE 2. MAXIMUM, MINIMUM, AND INCREMENTAL VALUES OF THE SCOUT-TARGET PARAMETERS

PARAMETER	MIN	MAX	INC
$x_s$	0	0	0
$y_s$	0	0	0
$z_s$	10 m	35. m	5. m
$\theta = \tan^{-1}(x_t/y_t)$	0°	90°	18°
$r = (x_t^2 + y_t^2)^{1/2}$	2000 m	4000 m	400 m
$z_t$	0	0	0
$H$	45°	45°	0
$E$	5°	10°	1°
$\phi$	-4°	1°	1°
$\alpha$	-45.89°	45.66°	--
$\varepsilon$	0.84°	11.72°	--
$R_{TS}$	2000.02 m	4000.15 m	--

Allowing five parameters to assume 6 values each yields  $6^5$  combinations or 7776 Scout-target states.

The resulting standard deviation about the target location were

$$\sigma_{E_x} = 67.26 \text{ meters}$$

$$\sigma_{E_y} = 67.26 \text{ meters}$$

$$\sigma_{E_z} = 26.87 \text{ meters}$$

and the Circular Probable Error (CEP) was

$$CEP = 1.1774\sigma_{E_x} = 79.19 \text{ meters}$$

In order to determine the contribution of individual errors to the total error, nine additional runs were made on the computer, each with all but one of the nine standard deviations zeroed. That is, the first run was made with  $e_x$  nonzero, the second with  $e_y$  nonzero, etc.

Letting  $\sigma_{E_x}^2(e_i)$  = variance of target x position with the  $i^{th}$  error nonzero, etc.

and  $\sigma_{E_x}^2$  = variance of target X position with all errors nonzero, etc.

Then

$V_i(E_x) = \frac{\sigma_{E_x}^2(e_i)}{\sigma_{E_x}^2}$  measure of the contribution of the  $i^{th}$  error to the total error in X.

The following table lists the results of the above calculations:

TABLE 3. CONTRIBUTION OF INDIVIDUAL ERRORS TO THE TOTAL ERROR

Nonzero Input Error (i)	$V_i(E_x)$ (%)	$V_i(E_y)$ (%)	$V_i(E_z)$ (%)
$e_x$	66.87	0	0
$e_y$	0	66.87	0
$e_z$	0	0	0.55
$e_H$	31.87	31.87	0
$e_\epsilon$	4.97 by $10^{-4}$	4.97 by $10^{-4}$	75.52
$e_\phi$	0.16	0.16	23.88
$e_\alpha$	4.08 by $10^{-3}$	4.08 by $10^{-3}$	3.46 by $10^{-4}$
$e_\varepsilon$	3.54 by $10^{-5}$	1.99 by $10^{-5}$	0.05
$e_{RTS}$	1.10	1.10	1.12 by $10^{-3}$

It is clear from Table 3 that, within the range of Scout-target state parameters chosen, the navigation errors ( $e_x$  and  $e_y$ ) and the heading error ( $e_h$ ) are the major contributors to the target location error in the x-y plane, which is the plane of the UTM grid. For this reason it was decided to take a more detailed look at the relationship between navigation error, heading error, and total error. Realizing that the contribution to the total error from heading error is dependent on range, six runs were made with all parameters and errors as in Tables 1 and 2 except that the parameter  $r$  was held constant throughout each run. The results of each run were then a function of  $r$  which assumed values ranging from 1,000 meters for the first run to 6,000 meters for the sixth run. Table 4 shows the results of those runs.

TABLE 4. CONTRIBUTION OF NAVIGATION AND HEADING ERRORS TO TOTAL ERROR

r meters	TOTAL ERROR (m) ( $\sigma_{EX} = \sigma_{EY}$ )	ERROR DUE TO NAVIGATION		ERROR DUE TO HEADING	
		ABSOLUTE (meters)	RATIO OF VARIANCES	ABSOLUTE (meters)	RATIO OF VARIANCES
1,000	56.82	55.0	93.69%	12.34	4.72%
2,000	60.72	55.0	82.04%	24.68	16.52%
3,000	66.73	55.0	67.93%	37.02	30.78%
4,000	74.32	55.0	54.77%	49.37	44.13%
5,000	83.07	55.0	43.84%	61.71	55.18%
6,000	92.65	55.0	35.24%	74.05	63.88%

Finally, 120 additional runs were made on the computer each with a fixed  $r$ , navigation error, and heading error. The intent of these runs was to show the trade-off between heading error and navigation error for varying ranges. The results are listed in Tables 5 through 10.

TABLE 5. TRADE-OFF BETWEEN HEADING ERROR AND NAVIGATION  
ERROR FOR RANGE EQUAL TO 1,000 METERS

Navigation Error CEP (meters)	Resulting Tgt Loc CEP			
	Heading Errors ( $^{\circ}$ )			
	0.5	1.0	1.5	2.0
1,000	1000.02	1000.11	1000.24	1000.43
500	500.16	500.32	500.58	500.96
250	250.21	250.53	251.04	251.79
100	100.57	101.36	102.66	104.43
50	51.25	52.78	55.22	58.47

TABLE 6. TRADE-OFF BETWEEN HEADING ERROR AND NAVIGATION  
ERROR OR RANGE EQUAL TO 2,000 METERS

Navigation Error CEP (meters)	Resulting Tgt Loc CEP			
	Heading Errors ( $^{\circ}$ )			
	0.5	1.0	1.5	2.0
1,000	1000.11	1000.43	1000.95	1001.68
500	500.32	500.96	502.00	503.48
250	250.52	251.79	253.88	256.78
100	101.37	104.46	109.39	115.95
50	52.81	58.51	66.92	77.18

TABLE 7. TRADE-OFF BETWEEN HEADING ERROR AND NAVIGATION  
ERROR FOR RANGE EQUAL TO 3,000 METERS

Navigation Error CEP (meters)	Resulting Tgt Loc CEP			
	Heading Errors ( $^{\circ}$ )			
	0.5	1.0	1.5	2.0
1,000	1000.25	1000.95	1002.14	1003.80
500	500.60	502.02	504.37	507.66
250	251.07	253.89	258.52	264.88
100	102.69	109.42	119.78	132.94
50	55.30	66.96	82.82	100.91

TABLE 8. TRADE-OFF BETWEEN HEADING ERROR AND NAVIGATION  
ERROR FOR RANGE EQUAL TO 4,000 METERS

Navigation Error CEP (meters)	Resulting Tgt Loc CEP			
	Heading Errors ( $^{\circ}$ )			
	0.5	1.0	1.5	2.0
1,000	1000.42	1001.70	1003.80	1006.74
500	500.97	503.49	507.67	513.46
250	251.82	256.80	264.89	275.83
100	104.52	115.0	132.96	153.59
50	58.60	77.25	100.95	126.90

TABLE 9. TRADE-OFF BETWEEN HEADING ERROR AND NAVIGATION  
ERROR FOR RANGE EQUAL TO 5,000 METERS

Navigation Error CEP (meters)	Resulting Tgt Loc CEP			
	Heading Errors ( $^{\circ}$ )			
	0.5	1.0	1.5	2.0
1,000	1000.67	1002.65	1005.93	1010.52
500	501.45	505.39	511.87	520.82
250	252.78	260.49	272.86	289.29
100	106.80	123.96	148.20	176.63
50	62.60	88.75	120.32	153.99

TABLE 10. TRADE-OFF BETWEEN HEADING ERROR AND NAVIGATION  
ERROR FOR RANGE EQUAL TO 6,000 METERS

Navigation Error CEP (meters)	Resulting Tgt Loc CEP			
	Heading Errors ( $^{\circ}$ )			
	0.5	1.0	1.5	2.0
1,000	1000.97	1003.82	1008.54	1015.11
500	502.04	507.69	516.96	529.68
250	253.94	264.93	282.29	304.95
100	109.53	133.03	164.93	201.25
50	67.16	101.04	140.40	181.70

## 5. CONCLUSIONS

Within the context of this report, it is clear that the navigation and heading system errors have the most significant impact upon target location error. This is demonstrated by the data presented in Tables 3 and 4. Tables 5 through 10 demonstrate that, for large navigation errors, the target location CEP is relatively insensitive to changes in heading errors and changes in range. For smaller navigation errors, the contribution from heading errors becomes increasingly significant for increasing range. As seen in Table 10, for instance, with  $r = 6,000$  meters and navigation error = 50 meters, the target location error increases from 67.16 meters to 181.7 meters (a factor of 2.7) as the heading error increases from  $0.5^{\circ}$  to  $2^{\circ}$ .

Finally, it should be emphasized that when using the results presented in this report, one must be mindful of the fact that all system errors, including navigation and heading, have been assumed to be independent when, in the real world, this may not be the case.

## 6. ACKNOWLEDGEMENT

The author wishes to express his graditude to Dr. R. J. Niemela for his valuable assistance during the performance of this study.

## APPENDIX

## PROGRAM LISTINGS AND SAMPLE OUTPUT

```

COMMON XS,YS,ZS,XT,YT,ZT,H,E,PHI,ALPHA,EPS,RTS,PD(3,9),IXYZ
DIMENSION AMEAN(9)
DIMENSION VINIT(9),VFINAL(9),VINC(9),ERROR(9),NI(9)
DIMENSION ERINIT(9),ERFIN(9),ERINC(9),MI(9)
EQUIVALENCE (NI(1),N1),(NI(2),N2),(NI(3),N3),(NI(4),N4),(NI(5),N5)
1,(NI(6),N6),(NI(7),N7),(NI(8),N8),(NI(9),N9),(MI(1),M1),(MI(2),M2)
2,(MI(3),M3),(MI(4),M4),(MI(5),M5),(MI(6),M6),(MI(7),M7),(MI(8),M8)
3,(MI(9),M9)
DOUBLE PRECISION SMSIGX,SMSIGY,SMSIGZ,SUMEMX,SUMEMY,SUMEMZ
LOGICAL CREG
READ(3,5) IXYZ,NLINES,ITEST
5 FORMAT(I1,2I3)
READ(3,10) (AMEAN(I),I=1,9)
READ(3,12) (ERINIT(I),I=1,9)
READ(3,10) (ERFIN(I),I=1,9)
READ(3,10) (ERINC(I),I=1,9)
10 FORMAT(3F6.1,6(1PE10.3))
READ(3,10) (VINIT(I),I=1,9)
READ(3,10) (VFINAL(I),I=1,9)
READ(3,10) (VINC(I),I=1,9)
WRITE(1,11)
11 FORMAT(1X,45HSET CREG(1) FOR POLAR X-Y TARGET COORDINATES.)
PAUSE
NTEST=0
PD(1,1)=1.0
PD(1,2)=0.0
PD(1,3)=0.0
PD(2,1)=0.0
PD(2,2)=1.0
PD(2,3)=0.0
PD(3,1)=0.0
PD(3,2)=0.0
PD(3,3)=1.0
PD(3,4)=0.0
DO 20 I=1,9
IF(ABS(ERINC(I)) .GT. 1.0E-20) GO TO 12
MI(I)=1
GO TO 13
12 MI(I)=ABS((ERFIN(I)-ERINIT(I))/ERINC(I))+1.5
13 IF(ABS(VINC(I)) .GT. 1.0E-20) GO TO 15
NI(I)=1
GO TO 20
15 NI(I)=ABS((VFINAL(I)-VINIT(I))/VINC(I))+1.5
20 CONTINUE
CON=3.14159/180.0
KOUNT=0
DO 30 IE1=1,M1
ERROR(1)=ERINIT(1)+FLOAT(IE1-1)*ERINC(1)
DO 30 IE2=1,M2
ERROR(2)=ERINIT(2)+FLOAT(IE2-1)*ERINC(2)
DO 30 IE3=1,M3
ERROR(3)=ERINIT(3)+FLOAT(IE3-1)*ERINC(3)

```

```

DO 30 IE4=1,M4
ERROR(4)=(ERINIT(4)+FLOAT(IE4-1)*ERINC(4))*CON
DO 30 IE5=1,M5
ERROR(5)=(ERINIT(5)+FLOAT(IE5-1)*ERINC(5))*CON
DO 30 IE6=1,M6
ERROR(6)=(ERINIT(6)+FLOAT(IE6-1)*ERINC(6))*CON
DO 30 IE7=1,M7
ERROR(7)=ERINIT(7)+FLOAT(IE7-1)*ERINC(7)
DO 30 IE8=1,M8
ERROR(8)=ERINIT(8)+FLOAT(IE8-1)*ERINC(8)
DO 30 IE9=1,M9
ERROR(9)=ERINIT(9)+FLOAT(IE9-1)*ERINC(9)
22 NSTATE=0
SUMEMX=0.0
SUMEMY=0.0
SUMEMZ=0.0
SMSIGX=0.0
SMSIGY=0.0
SMSIGZ=0.0
ALPMIN=1.0E+6
ALPMAX=0.0
EPSMIN=1.0E+6
EPSMAX=0.0
RNGMIN=1.0E+6
RNGMAX=0.0
DO 35 I=1,N1
XS=VINIT(1)+FLOAT(I-1)*VINC(1)
DO 35 J=1,N2
YS=VINIT(2)+FLOAT(J-1)*VINC(2)
DO 35 K=1,N3
ZS=VINIT(3)+FLOAT(K-1)*VINC(3)
DO 35 L=1,N4
H=(VINIT(4)+FLOAT(L-1)*VINC(4))*CON
HP=H/CON
DO 35 M=1,N5
E=(VINIT(5)+FLOAT(M-1)*VINC(5))*CON
EP=E/CON
DO 35 N=1,N6
PHI=(VINIT(6)+FLOAT(N-1)*VINC(6))*CON
PHIP=PHI/CON
DO 35 II=1,N7
XT=VINIT(7)+FLOAT(II-1)*VINC(7)
IF(IXYZ .EQ. 0) ALPHA=XT*CON
DO 35 JJ=1,N8
YT=VINIT(8)+FLOAT(JJ-1)*VINC(8)
IF(CREG(1)) XT=(VINIT(7)+FLOAT(II-1)*VINC(7))*SIN(YT*CON)
IF(CREG(1)) YT=(VINIT(7)+FLOAT(II-1)*VINC(7))*COS(YT*CON)
IF(IXYZ .EQ. 0) EPS=YT*CON
DO 35 KK=1,N9
ZT=VINIT(9)+FLOAT(KK-1)*VINC(9)
IF(IXYZ .EQ. 0) RTS=ZT
NSTATE=NSTATE+1

```

```

CALL GETPD
ALPHAP=ALPHA/CON
EPSP=EPS/CON
EX=0.0
EY=0.0
EZ=0.0
EMX=0.
EMY=0.
EMZ=0.
DO 25 LL=1,9
EX = EX+PD(1,LL)*PD(1,LL)*ERROR(LL)*ERROR(LL)
EY = EY+PD(2,LL)*PD(2,LL)*ERROR(LL)*ERROR(LL)
EZ = EZ+PD(3,LL)*PD(3,LL)*ERROR(LL)*ERROR(LL)
EMX = EMX+PD(1,LL)*AMEAN(LL)
EMY = EMY+PD(2,LL)*AMEAN(LL)
25 EMZ = EMZ+PD(3,LL)*AMEAN(LL)

SMSIGX=SMSIGX+EX
SMSIGY=SMSIGY+EY
SMSIGZ=SMSIGZ+EZ
SUMEMX=SUMEMX+EMX
SUMEMY=SUMEMY+EMY
SUMEMZ=SUMEMZ+EMZ
IF(ALPHAP .LT. ALPMIN) ALPMIN=ALPHAP
IF(ALPHAP.GT. ALPMA) ALPMA=ALPHAP
IF(EPSMIN) EPSMIN=EPSP
IF(EPSMAX) EPSMAX=EPSP
IF(RTS .LT. RNGMIN) RNGMIN=RTS
IF(RTS .GT. RNGMAX) RNGMAX=RTS
35 CONTINUE
STATE=SQRT(FLOAT(NSTATE))
EMX=SUMEMX/STATE
EMY=SUMEMY/STATE
EMZ=SUMEMZ/STATE
SIGX=DSQRT(SMSIGX)/STATE
SIGY=DSQRT(SMSIGY)/STATE
SIGZ=DSQRT(SMSIGZ)/STATE
DO 55 I11=4,6
55 ERROR(I11)=ERROR(I11)/CON
WRITE(6,100)
100 FORMAT(1H1,41X,44HI N P U T M A T R I X P A R A M E T E R S ///
C18X,14HSCOUT POSITION,16X,15HTARGET POSITION,5X,7HHEADING,5X,5HPIT
CCH,6X,4HROLL,2X,8HREL. AZ.,2X,8HREL. EL.,5X,5HRANGE/21X,8H(METERS)
C,21X,8H(METERS),11X,5H(DEG),6X,5H(DEG),5X,5H(DEG),3X,5H(DEG),5X,5H
C(DEG),9X,3H(M)//13X,2HXS,8X,2HYS,8X,2HZS,8X,1HR,5X,5HTHETA,8X,2HZT
C//)
WRITE(6,110) (VINIT(IJK),IJK=1,3),(VINIT(IJK),IJK=7,9),(VINIT(IJK)
C,IJK=4,6),ALPMIN,EPSMIN,RNGMIN
110 FORMAT(1X,3HMIN,1X,12F10.2)
WRITE(6,112) (VFINAL(IJK),IJK=1,3),(VFINAL(IJK),IJK=7,9),(VFINAL(I
CJK),IJK=4,6),ALPMA,EPSMAX,RNGMAX
112 FORMAT(1X,3HMAX,1X,12F10.2)
WRITE(6,114) (VINC(IJK),IJK=1,3),(VINC(IJK),IJK=7,9),(VINC(IJK),I

```

```
CJK=4,6)
114 FORMAT(1X,3HINC,1X,9F10.2,3(6X,4H****))
      WRITE(6,116) NSTATE
116 FORMAT(/1X,19HNUMBER OF STATES = ,I7//)
      WRITE(6,120)
120 FORMAT(46X,41HI N P U T E R R O R S T A T I S T I C S///32X,16H
      CSCOUT NAVIGATION,4X,3HALT,3X,7HHEADING,5X,5HPITCH,6X,4HROLL,2X,8HR
      CEL. AZ.,2X,8HREL. EL.,5X,5HRANGE/36X,8H(METERS),8X,3H(M)      ,4X,5H
      C(DEG),6X,5H(DEG),5X,5H(DEG),3X,5H(RAD),5X,5H(RAD),4X,8H(METERS),/3
      C4X,1HX,9X,1HY//)
      WRITE(6,130) (AMEAN(IJK),IJK=1,9) ,(ERROR(IJK),IJK=1,9)
130 FORMAT(6X,4HMEAN,15X,6F10.1,2F10.4,F10.1/6X,18HSTANDARD DEVIATION,
      C1X,6F10.1,2F10.4,F10.1////)
      WRITE(6,140)
140 FORMAT(16X,88HCOMPUTED ERROR STATISTICS FOR RESULTING DENSITY FUNC
      TIONS ABOUT TARGET X,Y,Z COORDINATES//38X,7HX ERROR,3X,7HY ERROR,3
      2X,7HZ ERROR//)
      WRITE(6,150) EMX,EMY,EMZ,SIGX,SIGY,SIGZ
150 FORMAT(15X,4HMEAN,16X,3F10.2/15X,18HSTANDARD DEVIATION,2X,3F10.2/1
      1H1)
      DO 155 I22=4,6
155 ERROR(I22)=ERROR(I22)*CON
30  CONTINUE
      CALL EXIT
      END
```

```

SUBROUTINE GETPD
COMMON XS,YS,ZS,XT,YT,ZT,H,E,PHI,ALPHA,EPS,RTS,PD(3,9),IXYZ
SH=SIN(H)
CH=COS(H)
SE=-SIN(E)
CE=COS(E)
SPHI=SIN(PHI)
CPHI=COS(PHI)
IF(IXYZ .EQ. 0) GO TO 40
XTS=XT-XS
YTS=YT-YS
ZTS=ZT-ZS
RTS=SQRT(XTS*XTS+YTS*YTS+ZTS*ZTS)
T1=(-CH*SE*CPHI+SH*SPHI)*YTS
T2=(SH*SE*CPHI+CH*SPHI)*XTS
SEPS=(T1-T2-CE*CPHI*ZTS)/RTS
CEPS=SQRT(1.0-SEPS*SEPS)
IF(ABS(CEPS) .GT. 1.0E-10) GO TO 10
EPS=1.570796
GO TO 20
10 EPS=ATAN2(SEPS,CEPS)
20 T1=-(CH*SE*SPHI+SH*CPHI)*YTS
T2=(SH*SE*SPHI-CH*CPHI)*XTS
SALPHA=(T1-T2-CE*SPHI*ZTS)/(RTS*CEPS)
CALPHA=SQRT(1.0-SALPHA*SALPHA)
IF(ABS(CALPHA) .GT. 1.0E-10) GO TO 30
ALPHA=1.570796
GO TO 50
30 ALPHA=ATAN2(SALPHA,CALPHA)
GO TO 50
40 SEPS=SIN(EPS)
CEPS=COS(EPS)
SALPHA=SINT(ALPHA)
CALPHA=COS(ALPHA)
50 T1=CE*CALPHA-SE*SPHI*SALPHA
T2=SE*CPHI*SEPS
T3=CE*SPHI*SALPHA-SE*CALPHA
T4=CE*CPHI*SEPS
T5=CPHI*CEPS*SALPHA-SPHI*SEPS
T6=-SPHI*CEPS*SALPHA-CPHI*SEPS
T7=-CE*SALPHA-SE*SPHI*CALPHA
T8=CPHI*CEPS*CALPHA
T9=CE*CALPHA-SE*SPHI*SALPHA
T10=SE*CPHI*CEPS
T11=-CPHI*SEPS*SALPHA-SPHI*CEPS
T12=CE*SPHI*CALPHA-SE*SALPHA
T13=CE*CPHI*CEPS
IF(IXYZ .EQ. 1) GO TO 60
XT=XS+RTS*(SH*(CEPS*T1-T2)+CH*T5)
YT=YS+RTS*(CH*(CEPS*T1-T2)-SH*T5)
ZT=ZS-RTS*(CEPS*T3+T4)
XTS=XT-XS

```

```
-- YTS=YT-YS  
ZTS=ZT-ZS  
60 PD(1,4)=YTS  
PD(1,5)=SH*ZTS  
PD(1,6)=RTS*(SH*(-SE*T5)+CH*T6)  
PD(1,7)=RTS*(SH*CEPS*T7+CH*T8)  
PD(1,8)=RTS*(SH*(-SEPS*T9-T10)+CH*T11)  
PD(1,9)=XTS/RTS  
PD(2,4)=-XTS  
PD(2,5)=CH*ZTS  
PD(2,6)=RTS*(CH*(-SE*T5)-SH*T6)  
PD(2,7)=RTS*(CH*(CEPS*T7)-SH*T8)  
PD(2,8)=RTS*(CH*(-SEPS*T9-T10)-SH*T11)  
PD(2,9)=YTS/RTS  
PD(3,5)=-(SH*XTS+CH*YTS)  
PD(3,6)=-RTS*CE*T5  
PD(3,7)=-RTS*CEPS*T12  
PD(3,8)=-RTS*(-SEPS*T3+T13)  
PD(3,9)=ZTS/RTS  
RETURN  
END
```

INPUT MATRIX PARAMETERS

	SCOUT POSITION (METERS)			TARGET POSITION (METERS)			HEADING (OEG)	PITCH (DEG)	ROLL (OEG)	REL. AZ. (DEG)	REL. EL. (OEG)	RANGE (M)
	X5	Y5	Z5	R	THETA	ZT						
MIN	0.00	0.00	10.00	2000.00	0.00	0.00	45.00	5.00	-4.00	-45.89	0.84	2000.02
MAX	0.00	0.00	35.00	4000.00	90.00	0.00	45.00	10.00	1.00	45.66	11.72	4000.15
INC	0.00	0.00	5.00	400.00	18.00	0.00	0.00	1.00	1.00	***	***	***

NUMBER OF STATES = 7776

INPUT ERROR STATISTICS

24

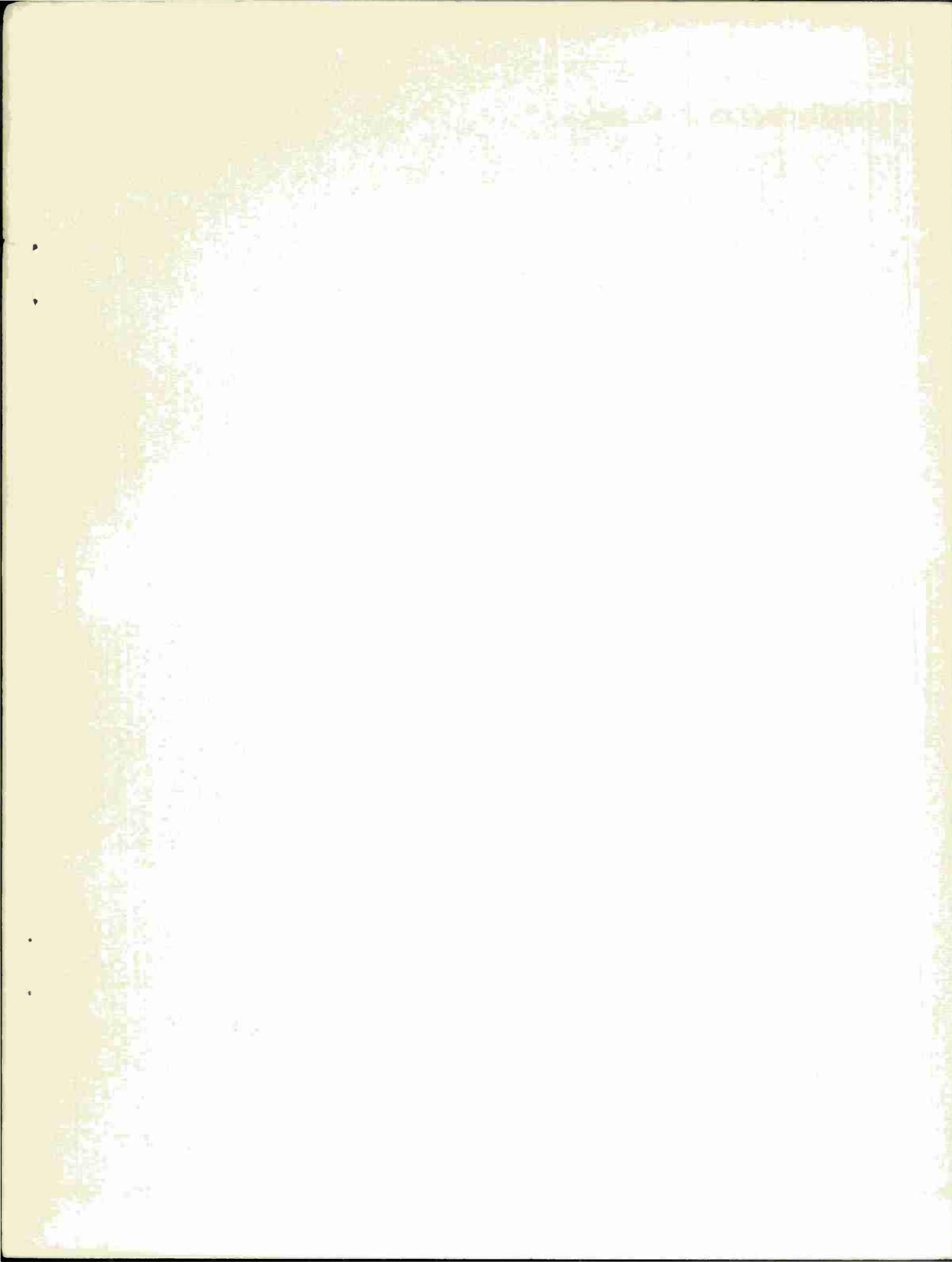
	SCOUT NAVIGATION (METERS)		ALT (M)	HEADING (OEG)	PITCH (OEG)	ROLL (DEG)	REL. AZ. (RAO)	REL. EL. (RAD)	RANGE (METERS)
	X	Y							
MEAN	0.0	0.0	0.0	0.0	0.0	0.0	0.0000	0.0000	0.0
STANDARD DEVIATION	55.0	55.0	2.0	1.0	0.5	0.5	0.0002	0.0002	10.0

COMPUTED ERROR STATISTICS FOR RESULTING DENSITY FUNCTIONS ABOUT TARGET X,Y,Z COORDINATES

X ERROR    Y ERROR    Z ERROR

MEAN	0.00	0.00	0.00
STANDARD DEVIATION	67.26	67.26	26.87

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